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Dirac Strings and Monopoles in the Continuum Limit of SU(2) Lattice Gauge Theory.

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Abstract

Magnetic monopoles are known to emerge as leading non-perturbative fluctuations in the lattice version of non-Abelian gauge theories in some gauges. In terms of the Dirac quantization condition, these monopoles have magnetic charge $|Q_M| = 2$. Also, magnetic monopoles with $|Q_M| = 1$ can be introduced on the lattice via the 't Hooft loop operator. We consider the $|Q_M| = 1, 2$ monopoles in the continuum limit of the lattice gauge theories. To substitute for the Dirac strings which cost no action on the lattice, we allow for singular gauge potentials which are absent in the standard continuum version. Once the Dirac strings are allowed, it turns possible to find a solution with zero action for a monopole–antimonopole pair. This implies equivalence of the standard and modified continuum versions in perturbation theory. To imitate the nonperturbative vacuum, we introduce then a nonsingular background. The modified continuum version of the gluodynamics allows in this case for monopoles with finite non-vanishing action. Using similar techniques, we construct the 't Hooft loop operator in the continuum and predict its behavior at small and large distances both at zero and high temperatures.

Introduction

While perturbative Yang-Mills theories appear to be understood beyond any doubt, non-perturbative physics is much more challenging at the moment. Moreover, the main source of knowledge in the non-perturbative domain is the lattice gauge theories. In particular, there exist rich data supporting the idea of the quark confinement through the magnetic monopole condensation (for review, see, e.g. [1]).

Any analytical treatment of magnetic monopoles in the continuum limit represents apparent difficulties because of singularities in the gauge potential A_μ. Indeed, such singularities are displayed already by the original Dirac monopole:

$$A_\mu dx_\mu = \frac{1}{2}\,(1+\cos\theta)\,d\varphi, \tag{1}$$

or in the component form in the spherical coordinates:

$$A_\theta = A_r = 0, \qquad A_\varphi = \frac{1}{2}\,\frac{(1+\cos\theta)}{r\sin\theta}\;. \tag{2}$$

The singularity along the line $\theta = 0$ represents the Dirac string, while the singularity at $r \to 0$ corresponds to a singular magnetic filed, $\mathbf{H} \sim \mathbf{r}/r^3$. In non-Abelian theories with Higgs mechanism the singularities are resolved and there exists the famous 't Hooft-Polyakov solution [2] with finite energy. In a particular gauge, the corresponding potential is given by

$$A^a_i \;=\; f(r)\frac{\varepsilon_{aik} r_k}{r^2} \tag{3}$$

where a is the color index, $a = 1, 2, 3$ and $f(r) \to 0$ as $r \to 0$ while $f(r) \to 1$ as $r \to \infty$.

In pure gauge theories, there are no monopole solutions with finite energy. To reconcile this with observation of monopoles on the lattice, one considers dual gauge theories which serve as infrared limit of QCD [3]. In its simplest version, the theory is build on an octet of dual gluons and three octets of scalar (Higgs) fields. In this paper, we would stick to consideration of monopoles within the fundamental QCD. The reason is that the monopoles on the lattice are defined beginning from elementary cubes, i.e. at smallest distances available. Our guiding principle is to reexamine the continuum limit by confronting the treatment of the monopole-associated singularities on the lattice and in the continuum.

In the lattice formulation, the singularities due to the Dirac string and at $r \to 0$ are treated differently. As was emphasized first by Polyakov [4], the Dirac strings are allowed, i.e. cost no action in the lattice compact U(1) theory. As for the $r \to 0$ singularity, it introduces in this case a physical divergence in the action. The suppression due to this divergence is overcome, however, by the entropy factor when the coupling constant g, included into the definition of A_μ above, is of order of unity.

In the non-Abelian gauge models the relation of monopoles to the action is much more obscure, as far as analytical results are concerned. Moreover, one of the most

important steps in introducing monopoles is a pure topological definition which makes no reference to the associated non-Abelian action [5]. In this formulation, monopoles are related to topology of gauge fixing. Namely, if the gauge is fixed (up to U(1) rotations) by directing a color vector h^a in, say, the third direction, then the fixation fails at the points where all the components h^a vanish. Moreover, one can prove that such points belong to monopole trajectories. The function of h^a can be played by any vector, for example, by a particular Lorenz component of the gluonic field-strength tensor, say, F^a_{12}. Vanishing of h^a has no direct effect on the action. Other Abelian projections revealing monopoles are also known, the most famous one seems to be the Maximal Abelian gauge (for review and references see [1]).

Monopoles which condense in the confining phase have magnetic charges $|Q_M| = 2$, the same as the 't Hooft-Polyakov monopole (3). In the corresponding U(1) projection the associated Dirac string does not introduce infinities because of the compactness of the U(1) subgroups of SU(2), see the discussion above. On the other hand the Dirac strings associated with $|Q_M| = 1$ monopoles are not allowed in the QCD vacuum since in the continuum limit they have infinite energy. However, one can introduce $|Q_M| = 1$ monopoles as external objects via the 't Hooft loop operator [6].

In this paper, we consider magnetic monopoles in the continuum provided that the continuum is understood as the limiting case of lattice theories. First, we generalize the treatment of Dirac strings within the lattice compact QED to the non-Abelian case. As expected, the lattice formulation of the non-Abelian theories corresponds to non-observability of the Dirac strings, defined in a particular way. To substitute for their effect in the continuum, one allows for certain singular potentials. Thus, we argue that the standard continuum formulation is to be modified in a certain way to allow for the Dirac strings.

It is amusing that once the Dirac strings are admitted into the continuum limit the $|Q_M| = 2$ monopoles cost no action either. Namely, we construct an explicit solution with zero action for a Dirac strings with open ends. In this respect the non-Abelian theories differ radically from their Abelian counterpart where the end points of the Dirac strings represent monopoles (1) with divergent action. It might worth emphasizing that the Abelian part of the fields in the no-action solution does correspond to the standard Abelian monopoles and it is the commutator term in the field-strength tensor which allows to nullify the non-Abelian action. This is in the correspondence with the instability of a single $|Q_M| = 2$ monopole with nonzero action in the non-Abelian pure gauge theory which is known since long [7].

The explicit monopole-pair solution with no action mentioned above is obtained in empty, or perturbative vacuum. We check that quantum fluctuations around this zero-action solution do not distinguish it from the perturbative vacuum either. Therefore the modified continuum version corresponding to the limit of the lattice theories brings no change in the perturbative domain as compared with the standard Lagrangian theory.

We then imitate non-perturbative vacuum of QCD by introducing nonsingular

background fields, $F^{soft}_{\mu\nu} \sim \Lambda^2_{QCD}$. Then there still exist Dirac strings with zero action whose color orientation is aligned with that of the background field. On the other hand, introduction of monopoles a la 't Hooft (see above) is related to some singular gauge transformations with their own color orientation. As a result, monopoles in the physical vacuum are associated, generally speaking, with an action of order $L \cdot \Lambda_{QCD}$ where L is the length of the monopole world trajectory.

Finally, the same techniques as used to construct invisible Dirac strings in the continuum limit produce a continuum analog for the 't Hooft loop operator. It shares the basic properties of the 't Hooft loop operator and allows to formulate new predictions for the intermonopole potential. At short distances the 't Hooft loop describes the Coulomb-like interaction of the monopoles with $|Q_M| = 1$, Ref. [8]. We fix the coefficient at front of this Coulombic term. At larger distances the $|Q_M| = 1$ monopoles, introduced via the 't Hooft loop interact with the $|Q_M| = 2$ monopoles of the medium. We describe this interaction within the effective Abelian Higgs model (for review and references see [1]), which uniquely fixes the Yukawa-like behavior of the intermonopole potential. We include also consideration of the 't Hooft loop at high temperatures where the Debye screening becomes essential.

The outline of the paper is as follows. In Section 1 we show that symmetries of the lattice and standard continuum actions of gluodynamics are different. We propose the modified continuum action which allows for the Dirac strings. In Sections 2, 3 monopole configurations within the new approach are considered. In Section 4 we introduce the 't Hooft loop operator in the continuum. In Section 5 the predictions for the 't Hooft loop are formulated. Our conclusions are summarized in the last section.

1 Dirac Strings in SU(2) Gauge Theory.

As is mentioned in the Introduction, the lattice formulation of the compact photodynamics gives a version of the U(1) gauge theory [4] with unobservable Dirac strings. In this section we develop a generalization of this construction to the case of SU(2) gauge model.

The general one–plaquette action of SU(2) lattice gauge theory (LGT) can be represented as:

$$S_{lat}(U) = \frac{4}{g^2} \sum_p S_P(1 - \frac{1}{2} \operatorname{Tr} U[\partial p]) \,, \tag{4}$$

where g is the bare coupling, ∂p is the boundary of an elementary plaquette p, the sum is taken over all p, $U[\partial p]$ is the ordered product of link variables U_l along ∂p. To have the correct naive continuum limit the function S_P should obey the condition $\lim_{x\to 0} S_P(x) = x + \cdots$. In particular, if $S_P(x) = x$ then (4) is the standard Wilson action. The exponent of the lattice field strength tensor F_p defines $U[\partial p]$:

$$U[\partial p] \; = \; e^{i\hat{F}_p} \; = \; \cos[\frac{1}{2}|F_p|] \; + i\tau^a n^a_p \; \sin[\frac{1}{2}|F_p|] \,, \tag{5}$$

where $\hat{F} = F^a \cdot \tau^a/2$, $|F| = \sqrt{F^a F^a}$ and we define $n^a_p = F^a_p/|F_p|$ for $|F_p| \neq 0$, n^a is an arbitrary unit vector for $|F_p| = 0$. Sometimes we also use the vector-like notations $\vec{F}$ instead of F^a. The lattice action (4) depends only on $\cos[\frac{1}{2}|F_p|]$. Therefore the action of the SU(2) LGT possesses not only the usual gauge symmetry, but allows also for the gauge transformations which shift the field strength by $4\pi k$, $|F_p| \to |F_p| + 4\pi k$, $k \in Z$:

$$\begin{aligned} e^{i\hat{F}_p} &= \exp\{ \, i|F_p| \, \hat{n}_p \, \} = \\ &= \exp\{ \, i(|F_p| + 4\pi) \, \hat{n}_p \, \} = \exp\{ \, i(F^a_p + 4\pi n^a_p) \, \tau^a/2 \, \} \,, \end{aligned} \tag{6}$$

Thus the symmetry inherent to the lattice formulation can be represented as:

$$F^a_p \;\to\; F^a_p + 4\pi n^a_p \,, \qquad \vec{F}_p \times \vec{n}_p = 0 \,, \qquad \vec{n}^2_p = 1 \,. \tag{7}$$

The symmetry (7) is absent in the conventional continuum action, $\int (F^a_{\mu\nu})^2 \, d^4x$ and therefore the continuum limit of SU(2) LGT is different from the commonly accepted SU(2) gluodynamics at least in this respect. Below we explore the consequences of Eq. (7) for the continuum theory.

In the continuum limit n^a_p becomes a singular two-dimensional structure ${}^*\Sigma^a_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}\Sigma^a_{\lambda\rho}$ which is a generalization of the Dirac strings in the compact electrodynamics and which transforms in the adjoint representation of the gauge group. Consider first a special class of the gauge potentials which may be gauge transformed to pure Abelian fields, $A^a_\mu = \delta^{a,3} A_\mu$. For such fields the action of the SU(2) gluodynamics coincides with the action of the compact U(1) gauge model, up to the ghost terms. Therefore in this gauge $\Sigma^a_{\mu\nu} = \delta^{a,3}\Sigma_{\mu\nu}$, where $\Sigma_{\mu\nu}$ is nothing else but the Dirac string:

$$\Sigma_{\mu\nu} = \int d^2\sigma \sqrt{g} \, t_{\mu\nu}(\sigma) \, \delta^{(4)}(x - \tilde{x}(\sigma)) \,, \tag{8}$$

with the world-sheet coordinates $\tilde{x}(\sigma)$ parameterized by σ_α, $\alpha = 1, 2$:

$$t_{\mu\nu}(\sigma) = \frac{1}{\sqrt{g}} \, \varepsilon^{\alpha\beta} \, \partial_\alpha \tilde{x}_\mu \, \partial_\beta \tilde{x}_\nu \,, \qquad t^2_{\mu\nu} = 2 \,, \qquad g(\sigma) = \mathrm{Det}[\, \partial_\alpha \tilde{x}_\mu \, \partial_\beta \tilde{x}_\mu \,] \,. \tag{9}$$

Thus for general gauge potentials

$$\Sigma^a_{\mu\nu} = \int d^2\sigma \sqrt{g} \, t^a_{\mu\nu}(\sigma) \, \delta^{(4)}(x - \tilde{x}(\sigma)) \,. \tag{10}$$

The second equality in (7) requires that

$$\vec{t}_{\mu\nu}(\sigma) \times {}^*\vec{F}_{\mu\nu}(\tilde{x}) = 0 \,, \tag{11}$$

where the continuum field strength tensor $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - i[\hat{A}_\mu, \hat{A}_\nu]$. Eq. (11) determines the color structure of $t^a_{\mu\nu}$:

$$t^a_{\mu\nu}(\sigma) \;=\; t_{\mu\nu}(\sigma) \, n^a(\sigma) \,, \qquad n^a(\sigma) \;=\; (t \cdot {}^*F^a) \left[(t \cdot {}^*F^b)^2 \right]^{-1/2} \,, \tag{12}$$

where $(t \cdot F^a) \equiv t_{\mu\nu}(\sigma)\, F^a_{\mu\nu}(\tilde{x})$ and n^a is normalized as $\vec{n}^2 = 1$. On the set of points where $(t \cdot {}^*F^a) = 0$ the direction of $n^a(\sigma)$ is arbitrary. Therefore, in the general case $\Sigma^a_{\mu\nu}$ is given by

$$\begin{aligned}\Sigma^a_{\mu\nu} &= \int d^2\sigma\, \sqrt{g}\, t_{\mu\nu}(\sigma)\, n^a(\sigma)\, \delta^{(4)}(x - \tilde{x}(\sigma))\,, \\ \vec{n}^{\,2}(\sigma) &= 1\,, \qquad \vec{n}(\sigma) \times (t_{\mu\nu}(\sigma)\, {}^*\vec{F}_{\mu\nu}(\tilde{x})) \;=\; 0\,.\end{aligned} \tag{13}$$

and the continuum analog of the lattice symmetry Eq. (6,7) is:

$$F^a_{\mu\nu} \;\to\; F^a_{\mu\nu} + 4\pi\, {}^*\Sigma^a_{\mu\nu}\,, \tag{14}$$

Note that we do not claim that the only string-like singularities which may exist in the continuum limit of SU(2) LGT are of the type (13,14). Indeed, there are known examples of various Abelian gauges (see [1] and references therein) in which Abelian monopoles and Dirac strings naturally arise. String singularities in these gauges are of the type (13), but their color orientation is different. Therefore the strings (13) are not the most general. Nevertheless, we claim that only the strings (13) produce no additional action. In other words the action of the SU(2) LGT in the continuum limit calculated with $F^a_{\mu\nu}$ and $F^a_{\mu\nu} + 4\pi^*\Sigma^a_{\mu\nu}$ is the same only if $\Sigma^a_{\mu\nu}$ is given by (13). We shall come back to discuss this issue in Section 3.

The action of SU(2) gluodynamics which possesses the additional symmetry (14) can be formally represented as:

$$Z = \int \mathcal{D}A\; \exp\Bigl\{-S(F)\Bigr\}\,, \tag{15}$$

$$S(F) = -\log\, \int \mathcal{D}\Sigma \exp\Bigl\{-\frac{1}{4g^2}\int d^4x\, \Bigl[F^a_{\mu\nu} + 4\pi\, {}^*\Sigma^a_{\mu\nu}\Bigr]^2\Bigr\}\,, \tag{16}$$

where the integration is over all possible surfaces (13). The expressions (15,16) are only formal since, as we show in Section 2 it is impossible to separate rigorously the measure $\mathcal{D}\Sigma$ from the gauge degrees of freedom in $\mathcal{D}A$. Nevertheless, the Eq. (15,16) is a good starting point for the analysis of the next section. Note that the action (16) is invariant under smooth SU(2) gauge transformations since vector n^a transforms in the same way as $F^a_{\mu\nu}$ does. By construction, this action is also invariant under transformations (13,14) which correspond to the lattice symmetry relations (6,7).

Note also that for self-intersecting surface $\Sigma_{\mu\nu}$, Eq. (13), the world-sheet vector field $n^a(\sigma)$ is generally multi-valued as function of $\tilde{x}$. Furthermore, for the non-orientable surfaces the field $n^a(\sigma)$ cannot be defined smoothly everywhere on Σ. To avoid these complications we consider only the orientable surfaces without self-intersections. This reservation is specific for Dirac strings in the non-Abelian case.

2 Monopoles.

We proceed now to consider Dirac strings with open ends. The end points can be associated, as usual, with monopoles. If one follows only the Abelian-like part of the field strength tensors, these are standard Dirac monopoles. However, the full non-Abelian action is no longer bounded in terms of the Abelian magnetic field and we will present a zero-action solution for open Dirac strings. Therefore contrary to the Abelian models the open Dirac strings in SU(2) gluodynamics are the gauge copies of the vacuum $A = 0$. Moreover, we show that this result is also valid at the one loop level and hence the Dirac strings (13) do not change the perturbation theory.

2.1 String Independence.

Consider the partition function (15,16) in case of a single surface $\Sigma^a_{\mu\nu}$:

$$Z[\Sigma] = \int \mathcal{D}A \exp\Big\{-\frac{1}{4g^2}\int d^4x \left[F^a_{\mu\nu} + 4\pi q\, {}^*\Sigma^a_{\mu\nu}\right]^2\Big\}\,, \tag{17}$$

where the constant q is equal to unity in (15,16). Varying the gauge fields A we get the classical equations of motion:

$$D_\nu \left(\hat F_{\mu\nu}(A) + 4\pi q\, {}^*\hat\Sigma_{\mu\nu}\right) = 0\,, \tag{18}$$

which should be supplemented by Bianchi identities:

$$D_\nu {}^*\hat F_{\mu\nu} = 0\,. \tag{19}$$

Note that Eq. (18) is consistent with the covariant conservation of electric currents:

$$D_\mu D_\nu \hat F_{\mu\nu} = -4\pi q\, D_\mu D_\nu {}^*\hat\Sigma_{\mu\nu} \sim \int d^2\sigma_{\mu\nu}{}^*[D_\mu, D_\nu]\, \hat n(\sigma)\, \delta^{(4)}(x - \tilde x(\sigma)) = 0\,.$$

where the last equality is due to (13).

To appreciate the meaning of eq.(18) let us confine ourselves for the moment to the fields $A^a_\mu = Q^a \cdot A_\mu$ with a constant color direction Q^a. Then $n^a \sim Q^a$ and Eq. (18) becomes:

$$\partial_\nu \left(\partial_{[\mu} A_{\nu]}\right) = -4\pi q\, \partial_\nu {}^*\Sigma_{\mu\nu}\,. \tag{20}$$

The solution of this equation in the Landau gauge,

$$A^a_\mu = -Q^a \cdot 4\pi q\, \frac{1}{\Delta}\, \partial_\nu {}^*\Sigma_{\mu\nu}\,, \tag{21}$$

corresponds to the gauge potential of an Abelian monopole current $\partial\Sigma$ embedded into the SU(2) group. Thus $\Sigma_{\mu\nu}$ is the Dirac string worldsheet.

Let us show that the shape of Σ is irrelevant, that is the surface Σ can be shifted by a gauge transformation provided that the boundary $\partial\Sigma$ is fixed. Assuming that

the orientable surface (13) has no self-intersections, we may write $\Sigma^a_{\mu\nu} = n^a \Sigma_{\mu\nu}$. Consider then a closed non self-intersecting surface $\mathcal{S}$ on which the vector field $n^a(\sigma)$ is a single valued function of $\tilde{x}$. We can define the field $n^a(x)$, $\vec{n}^{\,2} = 1$, in the whole space-time in such a way that

$$n^a(x) = n^a(\tilde{x}) \quad \text{for} \quad x \in \mathcal{S}\,. \tag{22}$$

Note that the definition of the vector $n^a(x)$ is not unique, but this is irrelevant for our analysis.

Consider the following gauge transformation matrix:

$$\Omega(\mathcal{V_S}) = \exp\{\, i\ \alpha(\mathcal{V_S}, x)\ \vec{n}(x)\vec{\tau}\ \}\,, \tag{23}$$

$$\alpha(\mathcal{V_S}, x) = 2\pi q \int\limits_{\infty}^{x} V_\mu dx_\mu\,, \qquad V_\mu = \int\limits_{\mathcal{V_S}} \left({}^*d^3\zeta\right)_\mu \delta^{(4)}(x-\zeta)\,, \tag{24}$$

where V_μ is a characteristic function of the volume $\mathcal{V_S}$ bounded by the surface $\mathcal{S}$. The first integral in (24) is taken along any path C_x connecting infinity with the point x. Under the general gauge transformation Ω the field strength tensor transforms as:

$$\hat{F}_{\mu\nu}(A^\Omega) = \Omega^+ \hat{F}_{\mu\nu}(A)\Omega + i\Omega^+ \left[\partial_\mu, \partial_\nu\right]\Omega = \Omega^+ \hat{F}_{\mu\nu}(A)\Omega + \hat{F}_{\mu\nu}(i\Omega^+\partial\Omega)\,. \tag{25}$$

Straightforward calculations show that for Ω defined by Eq. (23)

$$\begin{aligned} F^a_{\mu\nu}(i\Omega^+\partial\Omega) &= -2n^a\ [\partial_\mu, \partial_\nu]\alpha \\ &\quad - \Big(\sin[2\alpha]\ \delta^{ac} + (1-\cos[2\alpha])\varepsilon^{abc} n^b\Big)\ [\partial_\mu, \partial_\nu] n^c\,. \end{aligned} \tag{26}$$

Note that the function $\alpha(x)$ takes only two values, 0 and $2\pi q$. Therefore for integer or the half-integer valued charge q we have:

$$F^a_{\mu\nu}(i\Omega^+\partial\Omega) = -2\ n^a\ [\partial_\mu, \partial_\nu]\alpha = -4\pi q\ n^a\ \partial_{[\mu} V_{\nu]} = -4\pi q\ n^a\ {}^*\mathcal{S}_{\mu\nu}\,. \tag{27}$$

Thus the gauge transformation considered adds a closed surface ${}^*\mathcal{S}_{\mu\nu}$ to the field strength tensor, $\hat{F}(A^\Omega) = \Omega^+\hat{F}(A)\Omega - 4\pi q\ {}^*\hat{\mathcal{S}}$, $\hat{\mathcal{S}} = \hat{n}\mathcal{S}$. It is easy to see that the color structure of the surface $\Sigma^a_{\mu\nu}$ was inessential in our analysis. Indeed, one may perform exactly the same transformations with arbitrary $n^a(\sigma)$, $\vec{n}^{\,2} = 1$, instead of (13). Therefore the orientable non self-intersecting surface Σ in Eq. (17) with arbitrary color orientation can be deformed by the singular gauge transformation provided that $q = 0, \pm\frac{1}{2}, \pm 1, \ldots$.

We see that the situation looks similar to the Abelian case where the shape of the Dirac strings is inessential and can be changed by a gauge transformation so that only the end points of the strings have a physical meaning: they are identified with monopoles. However despite of this similarity the Yang–Mills theory is different in some respects. In particular, for the examples considered below the boundaries of the strings (13) have zero action thus being a pure gauge artifacts[1].

[1] Note that in the standard Yang–Mills theory these configurations have infinite action and therefore are not important.

2.2 Open Strings With Zero Action.

Consider the following gauge transformation matrix:

$$\Omega_1 = \begin{pmatrix} \cos\frac{\theta}{2}\, e^{i\varphi} & \sin\frac{\theta}{2} \\ & \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\, e^{-i\varphi} \end{pmatrix}, \qquad \Omega_1^+ \tau^3 \Omega_1 = \hat{x}^a \tau^a \,, \tag{28}$$

defined in the time-slice $t = 0$; θ, φ are polar and azimuthal angles. Performing the gauge transformation on the pure vacuum configuration $A = 0$ one gets:

$$^*F^a_{\mu\nu}(A^{\Omega_1}) = 4\pi \; \delta_{0,[\mu} \; \delta_{\nu],3} \; \delta^{a,3} \cdot \Theta(z)\delta(x)\delta(y) \,. \tag{29}$$

Therefore the singular gauge transformation (28) produces singular $F^a_{\mu\nu}$ as well, but the singularity in (29) is of allowed type (14) with time independent surface Σ directed along τ^3 in the color space. On the other hand, in the gauge transformed potentials one finds an Abelian monopole which is double charged in terms of the minimal Dirac quantization condition (cf. Eq. (1)):

$$\begin{array}{rcl} A^3 & = & A^3_\mu dx_\mu = -(1+\cos\theta)\; d\varphi \,, \\ A^+ & = & (A^1_\mu + iA^2_\mu) dx_\mu = -e^{i\varphi}\; (d\theta - i \sin\theta \; d\varphi) \,. \end{array} \tag{30}$$

The interpretation of (29,30) is as follows. In the U(1) case due to the magnetic flux conservation the Dirac string terminates at an Abelian monopole with the magnetic field $|\mathbf{H}| \sim 1/r^2$. In the SU(2) gauge model the Abelian string with net flux 4π may disappear into the vacuum. Although we still have the conservation of Abelian flux, this does not imply any bound on the action. In fact, because of the nontrivial components $A^\pm$ the full SU(2) action is zero. The zero action of the configuration (30) is due to the cancellation between the Abelian-like and commutator pieces in $F^a_{\mu\nu}$. Note that already in Ref. [7] it was shown that the double charged Abelian monopoles being immersed into SU(2) gauge group are unstable against fluctuations of non-Abelian components of gauge fields.

Proceed now to generalizing (28) to the case of finite Dirac string. Consider the potential A_μ which in U(1) theory represents the monopole–antimonopole pair located at $x, y = 0$, $z = \pm R/2$:

$$A_\mu dx_\mu = \frac{1}{2}\left(\frac{z_+}{r_+} - \frac{z_-}{r_-}\right)\; d\varphi = A_D(z,\rho) d\varphi \,, \qquad 0 \le A_D(z,\rho) \le 1 \tag{31}$$

$$z_\pm = z \pm R/2 \,, \qquad \rho^2 = x^2 + y^2 \,, \qquad r^2_\pm = z^2_\pm + \rho^2 \,, \tag{32}$$

and the following gauge transformation matrix

$$\Omega_2 = \begin{pmatrix} e^{i\varphi}\sqrt{A_D} & \sqrt{1-A_D} \\ & \\ -\sqrt{1-A_D} & e^{-i\varphi}\sqrt{A_D} \end{pmatrix} . \tag{33}$$

It is easy to check that (33) when applied to the vacuum $A = 0$ produces a string of the type (13,14) which begins and terminates at the points $\rho = 0$, $z = z_{\pm}$, respectively:

$$ {}^*F^a_{\mu\nu} = 4\pi\, \delta_{0,[\mu}\, \delta_{\nu],3}\, \delta^{a,3} \cdot \Theta(R/2 - |z|)\, \delta(x)\, \delta(y)\,. \tag{34} $$

The corresponding gauge potential $A = i\Omega_2^+ \partial \Omega_2$ contains Abelian monopole and antimonopole located at the ends of the string, $A^3_\mu dx_\mu = -2A_D(z,\rho)d\phi$.

The described above monopole configurations might be related to the monopoles common to the the lattice Abelian projections [1]. In a way, it is a consequence of the asymptotic freedom alone. Indeed, by the monopole one understands field configurations which in their Abelian part look like the standard Dirac monopole (1). The action associated with the Abelian monopole is linearly divergent in the ultraviolet,

$$ S_{Abelian} \sim (ag^2(a))^{-1}\,, $$

where a is an ultraviolet cut off, say, the lattice spacing. At first sight, on the background of this linear divergence the logarithmic behavior of the coupling is not important at all. However, it was shown in Ref. [4] that the a^{-1} factor in the action can be overcome by the entropy since it is proportional to an exponential of the length of monopole trajectories measured in the same units of a. As a result, the value of the coupling is becoming crucial and the Abelian-like monopoles can be abundant in the vacuum only if the coupling is of order unit, $g^2 \sim 1$. Which is inconsistent with the asymptotic freedom of the gluodynamics. The only way out is to have the non-Abelian field strength vanishing at short distances, $F^a_{\mu\nu} \to 0$ at $r \to 0$. In other words, the cancellation of the Abelian-like and commutator terms in the field strength tensor should be *exact* at short distances. The latter condition is satisfied by (30) which appears to be the unique monopole solution at short distances

The monopole structure at short distances can be studied directly on the lattice. At the distances available so far, the monopoles in SU(2) LGT are associated with a sizeable excess in the action, although the excess is substantially smaller than it would be in the pure Abelian case [9, 10]. Further measurements at smaller distances would be very interesting.

2.3 Quantum Corrections.

The examples presented above show that an arbitrary (non self-intersecting) string (13) may be considered as a result of combined gauge transformations of the type (23), (28), (33). Moreover, in the case of trivial background $\hat{F}(A) = 0$ such a singular gauge transformations are allowed and produce no action. A crucial question is whether the strings (13) are equivalent to gauge transformations when quantum fluctuations are included. Of course, if it were not so that the gauge transformations considered are singular, there would be no doubt that the quantum corrections do not destroy equivalence of the two field configurations related by a gauge transformation. But because of the presence of singularities we performed an explicit analysis of the

quantum corrections. The result is that the quantum corrections do not distinguish between the standard perturbative vacuum and the zero-action field configuration presented in the preceding section.

For the sake of definiteness we consider a straight Dirac string in the partition function (17)

$$Z[\Sigma] = \int \mathcal{D}A \exp\Big\{-\frac{1}{4g^2}\int d^4x \, \Big[F^a_{\mu\nu} + 4\pi \, {}^*\Sigma^a_{\mu\nu}\Big]^2\Big\}\,, \tag{35}$$

$$\Sigma^a_{\mu\nu} = -4\pi \; \delta_{0,[\mu} \; \delta_{\nu],3} \; \delta^{a,3} \cdot \Theta(z)\delta(x)\delta(y)\,. \tag{36}$$

The "classical" solution of the field equations is the pure gauge configuration (30) $A^{cl} = i\Omega^+\partial\Omega$ where Ω is given by (28) and the "classical" action is $S^{cl} = 0$. Expanding the action up to the second order in small perturbations $A = A^{cl} + a$ one finds that in the background gauge $D_\mu(A^{cl})a_\mu = 0$ Eq. (35) becomes:

$$Z[\Sigma] = \mathrm{Det}^{-1}[D^2(A^{cl})] \tag{37}$$

since in the present case the Pauli paramagnetic term is zero. With conventional normalization to the perturbative vacuum to vacuum amplitude the question whether the string $\Sigma^a_{\mu\nu}$ is relevant on quantum level, is equivalent to exploring the spectrum of the operator $D^2(A^{cl})$:

$$D^2(A^{cl}) \;=\; M_1 \;+\; M_2 \;+\; M_3 \tag{38}$$

$$M^{ab}_1 = \delta^{ab}\vec{\partial}^2 \qquad M^{ab}_2 = 2\varepsilon^{akb}\vec{A}^k\vec{\partial} \qquad M^{ab}_3 = \varepsilon^{akb}\vec{\partial}\vec{A}^k + \vec{A}^a\vec{A}^b - \delta^{ab}\vec{A}^k\vec{A}^k \tag{39}$$

where superscripts denote the color indices and vector notations are used for spatial components of A^{cl}. Using the explicit form (28) one finds that the non-zero elements of the antisymmetric matrix M_2 are

$$\begin{array}{lcl} M^{12}_2 & = & \frac{2}{r^2}\,\frac{1+\cos\theta}{\sin^2\theta}\;\partial_\varphi \\ M^{13}_2 & = & -\frac{2}{r^2}\,\left(\cos\varphi\;\partial_\theta - \frac{\sin\varphi}{\sin\theta}\;\partial_\varphi\right) \\ M^{23}_2 & = & -\frac{2}{r^2}\,\left(\sin\varphi\;\partial_\theta + \frac{\cos\varphi}{\sin\theta}\;\partial_\varphi\right) \end{array} \tag{40}$$

where θ and φ are the polar and azimuthal angles, respectively. In the same coordinate system the matrix M_3 is given by

$$M_3 = -\frac{2}{r^2}\,\frac{1+\cos\theta}{\sin^2\theta}\,\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\cos\varphi\;\sin\theta & -\sin\varphi\;\sin\theta & 1-\cos\theta \end{array}\right] \tag{41}$$

It is convenient to perform the transformation $D^2(A^{cl}) \to R\;D^2(A^{cl})\;R^{-1}$ where the matrix R transforms to the spherical basis:

$$R = \left[\begin{array}{ccc} \cos\varphi\sin\theta & \sin\varphi\sin\theta & \cos\theta \\ \cos\varphi\cos\theta & \sin\varphi\cos\theta & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{array}\right]. \tag{42}$$

One finds that in the new basis:

$$D^2(A^{cl}) = \vec{\partial}^{\,2} + \frac{1}{r^2} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{\sin^2\theta} & \frac{2}{\sin^2\theta}\partial_\varphi \\ 0 & -\frac{2}{\sin^2\theta}\partial_\varphi & -\frac{1}{\sin^2\theta} \end{bmatrix} . \tag{43}$$

Next, introduce

$$a_0 \;=\; a_r \qquad\qquad a_\pm \;=\; -\,\frac{i}{\sqrt{2}}\,(a_\theta \mp i a_\varphi)\,, \tag{44}$$

then the operator $D^2(A^{cl})$ becomes diagonal:

$$D^2(A^{cl}) \;=\; \vec{\partial}^{\,2} + \frac{1}{r^2}\cdot \mathrm{diag}\left[0\,,\; -\frac{1}{\sin^2\theta}(1-2i\partial_\varphi)\,,\; -\frac{1}{\sin^2\theta}(1+2i\partial_\varphi)\right] \tag{45}$$

Once $D^2(A^{cl})$ is brought to the diagonal form, the direct calculation shows that the spectrum of (45) is identical to that of free Laplacian $\vec{\partial}^{\,2}$. Therefore, the quantum fluctuations do not distinguish the string, Eq. (35,36), from the perturbative vacuum. Thus the modified theory (15,16) is perturbatively equivalent to the conventional gluodynamics.

3 Strings in General Background.

We have shown that in the perturbation theory both closed and open Dirac strings with arbitrary color orientation carry no action and are thus pure gauge artifacts. On one hand, this conclusion is welcome since it shows that the continuum limit as understood in this paper perturbatively is the same as the standard continuum limit. And, indeed, there are no doubts in the validity of the standard perturbation theory. On the other hand, if it were so that the singular fields admitted now into the continuum formulation are not associated with any action at all then the new formulation would be equivalent to the standard one.

In this section we address this issue on a non-perturbative level and imitate the non-perturbative fields by a smooth background. The crucial observation then is that only the Dirac strings with the proper color alignment (13) cost no action in the continuum limit, while the Dirac strings associated with the monopoles defined a la 't Hooft [5] do not satisfy this constraint.

Consider as an example the class of Abelian gauges of Ref. [5] which are defined by the requirement that some adjoint operator h^a is to be directed along τ^3 in the color space. This operator may be arbitrary in principle, but for the given h^a the remaining gauge freedom consists of U(1) rotations around τ^3:

$$\begin{gathered} \Omega^+ \hat{h}\Omega \;\to\; h^3\tau^3\,, \\ \Omega \;=\; \tilde{\Omega}\,H\,, \qquad\qquad \tilde{\Omega}\in G/U(1)\,, \qquad\qquad H\in U(1)\,. \end{gathered} \tag{46}$$

This gauge condition is not defined on the set of points where $h^a = 0$ which in four dimensions defines the monopole trajectory. Clearly enough, the Dirac strings associated with the monopoles are oriented along the third direction in the color space. Thus, there exist now two different directions in the color space determined by the background field and through the gauge fixation inherent to the definition of the monopoles.

Note that while the boundary of the string singularity is fixed by the equations $h^a = 0$, the actual position of the string may be changed with a suitable choice of H. Indeed, the gauge transformation (46) gives rise to the Dirac string:

$$\hat{F}(A^{\Omega}) \;=\; H^{+}\tilde{\Omega}^{+}\hat{F}(A)\tilde{\Omega}H + iH^{+}\left[\partial,\partial\right]H + iH^{+}\left(\tilde{\Omega}^{+}\left[\partial,\partial\right]\tilde{\Omega}\right)H. \qquad (47)$$

The both terms $H^{+}\left[\partial,\partial\right]H$ and $\tilde{\Omega}^{+}\left[\partial,\partial\right]\tilde{\Omega}$ are proportional to τ^3. Let us stress that the freedom to choose the U(1) matrix H allows to shift the position of the string in the ordinary space. Simultaneously, the background field is also transformed and one may not say that shifting the Dirac string brings no change in the action. However, since there is no spontaneous breaking of the color symmetry, the dependence on the position of the string drops off after integrating over all the background fields.

Note that similar considerations apply to the 't Hooft loop operator which we consider in the next section. Indeed, the definition of the 't Hooft loop operator as well as its value in a given background are string dependent. But the freedom to shift the position of the string in the path integral approach guarantees that no physical result depends on the string position.

It is amusing to note that the present considerations provides with a general framework to understand the correlation between instantons and monopoles which has been discussed in various contexts recently (see, e.g., [11]). Indeed, background fields in the physical vacuum are described realistically by instantons (see [12] for a review). On the other hand , monopoles, as is argued above, are meaningful only in the presence of background fields.

4 The 't Hooft Loop in the Continuum Limit.

In this Section we show that the construction presented above allows to define and study the properties of the 't Hooft loop operator in the path integral formalism.

The 't Hooft loop in SU(2) LGT with the one-plaquette action (4) has the following form:

$$H_{lat}(\Sigma_j) = \exp\Big\{\frac{4}{g^2}\sum_{p\in{}^*\Sigma_j}\Big[S_P\Big(1-\frac{1}{2}\,\mathrm{Tr}\,U[\partial p]\Big)-S_P\Big(1+\frac{1}{2}\,\mathrm{Tr}\,U[\partial p]\Big)\Big]\Big\}\,, \qquad (48)$$

where ${}^*\Sigma_j$ is the set of the plaquettes dual to the surface Σ_j with the boundary j. In the path integral formulation the 't Hooft loop effectively changes the sign of the plaquette variables $U[\partial p]$ belonging to ${}^*\Sigma_j$: $U[\partial p] \to -U[\partial p]$. To define the

't Hooft loop in the continuum we consider the path integral, Eq. (17), with an open orientable non self-intersecting surface $\Sigma^a_j = n^a \Sigma_j$, $\partial \Sigma_j = j$, multiplied by the Wilson loop $W_J(\mathcal{C})$:

$$\begin{aligned} Z(\mathcal{C}, \Sigma_j) &= \int \mathcal{D}A \exp\left\{ -\frac{1}{4g^2} \int d^4x \, \left[F^a_{\mu\nu} + 4\pi q \, {}^*\Sigma^a_{j\,\mu\nu} \right]^2 \right\} \cdot W_J(\mathcal{C}) \,, & (49) \\ W_J(\mathcal{C}) &= \mathrm{Tr}\, \mathrm{Pexp}\Big\{ i \oint\limits_{\mathcal{C}} T^a_J A^a_\mu dx_\mu \Big\} \,, & (50) \end{aligned}$$

where T^a_J are the generators of SU(2) in the representation J. It is convenient to use the following integral representation [13]:

$$W_J(\mathcal{C}) = \int \mathcal{D}\omega \exp\Big\{ iJ \oint\limits_{\mathcal{C}} \left[A^\omega_\mu \right]^3 dx_\mu \Big\} \,, \quad \left[A^\omega_\mu \right]^3 = \mathrm{Tr} \left[\tau^3 \, \omega^+ (A_\mu + i\partial_\mu) \omega \right] \,,$$

where the path integral is over all gauge transformations ω of the potential A on the contour $\mathcal{C}$. Therefore

$$Z(\mathcal{C}, \Sigma_j) = \int \mathcal{D}A \mathcal{D}\omega \; \exp\{ -S(A, \Sigma_j) + iJ \oint\limits_{\mathcal{C}} \left[A^\omega_\mu \right]^3 dx_\mu \} \,. \qquad (51)$$

The action $S(A, \Sigma_j)$ and the measure of integration $\mathcal{D}A$ are gauge invariant. The gauge transformation $A \; \to \; A^{\omega^{-1}}$ defined on $\mathcal{C}$ allows to factorize the integral $\mathcal{D}\omega$:

$$Z(\mathcal{C}, \Sigma_j) = \int \mathcal{D}\omega \; \cdot \; \int \mathcal{D}A \exp\{ -S(A, \Sigma_j) + iJ \oint\limits_{\mathcal{C}} A^3_\mu dx_\mu \} \,. \qquad (52)$$

The expression (52) has the following meaning: if there is no gauge fixing in the path integral (49) the Wilson loop may be calculated exactly by restricting the gauge potential A to diagonal U(1) subgroup of SU(2) [13].

Now we deform the surface Σ_j spanned on the contour j to another orientable non self-intersecting surface Σ'_j spanned on the same contour. We also consider the field $n'^a(\sigma)$ defined on Σ'_j according to (13). Then the closed surface $\mathcal{S} = \Sigma_j - \Sigma'_j$, which bounds the 3-volume $\mathcal{V}_{\mathcal{S}} = \mathcal{V}_{\Sigma_j - \Sigma'_j}$, has no self-intersection points and there exists a vector field $n^a(x)$, defined in the whole space–time,

$$\begin{aligned} n^a(x) = n^a(\tilde{x}) \quad \text{for} \quad x \in \Sigma_j \,, \\ n^a(x) = n'^a(\tilde{x}) \quad \text{for} \quad x \in \Sigma'_j \,. \end{aligned} \qquad (53)$$

In particular, $n^a(x)$ is defined on the contour $\mathcal{C}$ and there exists an SU(2) matrix $h \in \mathcal{C}$, such that:

$$\left[h \, \sigma^3 \, h^+ \right]^a = n^a(x) \qquad\qquad x \in \mathcal{C} \,. \qquad (54)$$

After the gauge transformation $A \; \to \; A^h$ Eq. (52) becomes:

$$Z(\mathcal{C}, \Sigma_j) = \int \mathcal{D}\omega \; \cdot \; \int \mathcal{D}A \exp\{ -S(A, \Sigma_j) + iJ \oint\limits_{\mathcal{C}} (n^a A^a + [ih^+ \partial h]^3) \qquad (55)$$

Consider now the additional gauge transformation $\Omega(\mathcal{V}_\mathcal{S})$ (23,24) with $\mathcal{V}_\mathcal{S} = \mathcal{V}_{\Sigma_j - \Sigma'_j}$. A straightforward calculation gives

$$n^a \left[A^\Omega_\mu\right]^a = n^a A^a_\mu - 4\pi q \, V_\mu \,. \tag{56}$$

As is shown in Section 2.1 for integer and half-integer charges q this gauge transformation shifts Σ_j to Σ'_j:

$$S(A^\Omega, \Sigma_j) = S(A, \Sigma'_j) \,. \tag{57}$$

For self–consistency of the theory, the Wilson loop is to be invariant under the gauge transformations. If we apply the transformation (56,57) to $Z(\mathcal{C}, \Sigma_j)$, see Eq. (49), we get:

$$\Omega(\mathcal{V}_\mathcal{S}) : \quad Z(\mathcal{C}, \Sigma_j) \to Z(\mathcal{C}, \Sigma_j - \mathcal{S}) \cdot e^{-i \, 4\pi q J \, \mathcal{L}(\mathcal{C}, \mathcal{S})} \,, \tag{58}$$

where $\mathcal{L}(\mathcal{C}, \mathcal{S})$ is the 4D linking number between the closed contour $\mathcal{C}$ and the closed surface $\mathcal{S}$:

$$\mathcal{L}(\mathcal{C}, \mathcal{S}) = \oint\limits_{\mathcal{S}} \left({}^* d^2\sigma\right)_{\mu\nu} \oint\limits_{\mathcal{C}} dx_\nu \partial_\mu \Delta^{-1}(\tilde{x}(\sigma) - x) \,. \tag{59}$$

Since $\mathcal{L} \in \mathbb{Z}$ and J takes integer and half–integer values the independence of the Wilson loop on the gauge transformations Ω implies the quantization condition:

$$q \in \mathbb{Z} \,. \tag{60}$$

This equation is a direct analog of the Dirac quantization condition in electrodynamics. Physically it means that the electrically charged particle introduced by Wilson loop does not scatter on the Dirac string $\mathcal{S}$.

Now we show that the 't Hooft loop operator $H(\Sigma_j)$ is given by:

$$H(\Sigma_j) = \exp\Big\{S(\hat{F}) - S(\hat{F} + 2\pi {}^*\hat{\Sigma}_j)\Big\} \,, \tag{61}$$

where the surface Σ^a_j is bounded by the contour j and is given by Eq. (13), the action S is defined by Eq. (16). Indeed, the transformation (58), when applied to the quantum average of the product of the fundamental, $J = 1/2$, Wilson loop and operator (61), gives:

$$< H(j, \Sigma_j) \, W_{1/2}(\mathcal{C}) > = < H(j, \Sigma'_j) \, W_{1/2}(\mathcal{C}) \cdot e^{i\pi \mathcal{L}(\mathcal{C}, \Sigma_j - \Sigma'_j)} > \,. \tag{62}$$

This formula proves that the operator H is the 't Hooft loop operator since it is in accordance with relations given in Refs. [6, 14].

5 Predictions for the 't Hooft loop.

In this Section we consider the rectangular $T \times R$ time-like contours j, Eq. (61), with $T \gg R$. Then the expectation value of the 't Hooft loop operator is

$$< H(\Sigma_j) > = < H(T, R) > \ \sim \ e^{-T V_{m\bar{m}}(R)} \,, \tag{63}$$

where by analogy with the Wilson loop we refer to the quantity $V_{m\bar{m}}(R)$ as to the intermonopole (monopole–antimonopole) potential. It is worth emphasizing that the potential $V_{m\bar{m}}$ corresponds to the $|Q_M| = 1$ monopoles while in Ref. [10] the monopole–antimonopole potential with the charge $|Q_M| = 2$ has been studied. These double charged monopoles are identified with the Abelian monopoles in Abelian projections.

Below we formulate predictions for the 't Hooft loop operator, Eq. (61), and its expectation value, Eq. (63). In particular, we show that the 't Hooft loop operator inserts the pair of $|Q_M| = 1$ monopoles which are pure Abelian in the Maximal Abelian gauge. This fact allows to fix the short distance asymptotic of the intermonopole potential. We argue then that this potential at larger distances at zero and high temperatures is of Yukawa type. We also find the screening mass in both cases and compare it with the masses measured on the lattice [15]. Our estimates turn to be in agreement with the numerical data.

5.1 Intermonopole Potential at Small Distances.

Consider the potential $V_{m\bar{m}}(R)$ at small distances for the monopole–antimonopole pair introduced by the operator $H(T,R)$. The definition (61) shows that we have enough gauge freedom to take $\Sigma^a_j = \delta^{a,3}\Sigma_j$ on the non self-intersecting surface Σ_j. Then at the classical level the solution of the corresponding equations of motion is [8]:

$$\begin{gathered} A^3_\mu dx_\mu = \frac{1}{2}\left(\frac{z_+}{r_+} - \frac{z_-}{r_-}\right)\, d\varphi\,, \qquad A^{1,2}_\mu = 0\,, \\ z_\pm = z \pm R/2\,, \qquad \rho^2 = x^2 + y^2\,, \qquad r^2_\pm = z^2_\pm + \rho^2\,, \end{gathered} \tag{64}$$

and represents the Abelian monopole-antimonopole pair separated by the distance R. Since the monopoles in (64) have minimal allowed magnetic charge $q = 1/2$ (see Section 2.1), at the classical level the intermonopole potential is given by:

$$V_{m\bar{m}}(R) \;=\; -\,\frac{\pi}{g^2\, R} \;=\; -\,\pi^2\,\beta\,\frac{1}{4\pi R}\,, \qquad \beta = \frac{4}{g^2}\,. \tag{65}$$

Note that the statement on the Coulombic nature of the intermonopole potential at short distances is well known [8, 15]. However, the fixation of the coefficient in front of $1/R$ is new, to the best of our knowledge[2].

Since the potential (65) was obtained for pure Abelian fields, we still have to prove that the general solution with minimal energy in SU(2) gluodynamics is indeed a gauge rotation of (64). A straightforward way to test the Eq. (65) is to investigate the problem numerically. We have calculated the expectation value of the 't Hooft loop in the standard SU(2) lattice gauge theory in the limit $\beta \to \infty$. Technically this limit is realized with the help of the so–called cooling procedure which was

[2] The same coefficient is derived in Ref. [30], which appeared on the day of submission of the present paper.

used to minimize the expectation value of the 't Hooft loop with respect to the classical lattice equations of motion. Our calculations have been performed on the three-dimensional 24^3 lattice with periodic boundary conditions, which is adequate to consider the static monopole–antimonopole pair. We minimized the 't Hooft loop operator, which creates static monopole and antimonopole separated by the distance R. We have fitted our data for the monopole–antimonopole potential by:

$$V^{lat.}_{m\bar{m}}(R) \;=\; -\,\pi^2\,\beta\,\Delta^{-1}_{lat.}(R)\,, \tag{66}$$

where $\Delta^{-1}_{lat.}(R)$ is the three-dimensional lattice Coulomb potential. Eq. (66) is the lattice regularization of the continuum expression (65). Note that the lattice and continuum potentials drastically differ from each other and this is of crucial importance in fitting the lattice data: the potential (66) is regular at $R = 0$ contrary to (65).

Our numerical calculations confirmed the behavior (66) with accuracy 2%. We also observed that after the cooling procedure the fields are Abelian up to a gauge transformation. In more detail, we found that in the Maximal Abelian gauge the gauge fields are diagonal and consist of the Abelian monopoles located at the boundary of the string Σ_j, Eq. (61). Therefore the classical limit of the state created by non-Abelian 't Hooft loop is the *Abelian* monopole–antimonopole pair.

Moreover, once the result (65) is established classically, the effect of the quantum corrections is also known on general grounds. Namely, the effect of the quantum corrections should be reduced to the replacement of the bare coupling by the running one, $g^2 \to g^2(R)$. Although the result is easy to guess, its derivation might look rather mysterious. Indeed, we have now both non-Abelian magnetic monopoles as external objects and ordinary gluons as virtual particles. At first sight we need both the standard and dual formulations of the gluodynamics to describe interaction both with magnetic and electric charges. While in case of U(1) gauge theories such a formulation is well known [17], it is absent in case of non-Abelian theories. Thus, we seem to know how the coupling runs although do not know, whose coupling is it!

We think that the resolution of the paradox is in the Abelian nature of the $|Q_M| = 1$ monopoles established above. Indeed, the classical considerations allow us to fix vertices, or the Lagrangian. The exact Abelian nature of the monopoles implies that once we choose an Abelian gauge fixing only neutral bosons (diagonal gluons) interact with the monopoles $|Q_M| = 1$. The charged vector bosons are still manifested through the loops. Thus, the situation is similar to the U(1) case with inclusion of the effect of virtual charged particles. As for the virtual monopoles, their effect can be neglected since the monopoles $|Q_M| = 1$ are infinitely heavy in the continuum limit. There is no much difficulty to deal with this problem and one can check that indeed the effect of the loops is the running of the coupling g^2. The details of the U(1) case can be found in the review in Ref. [17], see also the recent paper [16]. As for the perturbative calculations in non-Abelian theories in the Abelian projections, they can be found in Ref. [18]

5.2 Abelian Dominance and Intermonopole Potential.

Next we discuss the monopole–antimonopole potential at larger distances. The basic idea is to apply the Abelian Dominance hypothesis [19]. Indeed, as has been shown above the 't Hooft loop operator inserts the $|Q_M| = 1$ monopole pair in the vacuum of SU(2) gauge theory. Moreover, in the Maximal Abelian gauge these monopoles become a pure Abelian objects. Therefore it is natural to expect that in this particular gauge the dominant contribution to the potential (63) is due to the interaction with Abelian fields. In the Maximal Abelian gauge the vacuum of zero temperature SU(2) gluodynamics is a dual superconductor where, instead of condensate of Cooper pairs, there exists a monopole condensate. The principle of Abelian Dominance assumes that long distance properties of gluodynamics might be explained in terms of the interaction with the monopole condensate (for reviews see, e.g., [1]).

Following this logic, we expect that at the zero temperatures the monopole–antimonopole potential is:

$$V_{m\bar{m}}(R) \;=\; -\,\frac{\pi}{g^2}\,\frac{e^{-\mu R}}{R} \tag{67}$$

$$V^{lat.}_{m\bar{m}}(R) \;=\; -\,\beta\,\pi^2\,(-\Delta+\mu^2)^{-1}_{lat.}(R) \tag{68}$$

where μ is the dual photon mass m_V and $(-\Delta+\mu^2)^{-1}_{lat.}$ is the three-dimensional lattice Yukawa potential. The recent numerical investigation of the 't Hooft loop in SU(2) lattice gauge theory [15] agrees with Eq. (67). The value of $\mu \approx 3.24(42)\sqrt{\sigma}$ obtained in Ref. [15] is quite close to the dual photon mass $m_V \approx 1$ GeV $= 2.3\sqrt{\sigma}$ found in Ref. [20]. Let us also note that we would not identify directly the mass μ in Eq. (67) with a glueball mass. Indeed, the definition of the 't Hooft loop is highly nonlocal and includes a Dirac string with infinite action. Therefore, the validity of the dispersive relations is questionable in this case. Note, however, that μ in Eq. (67) coincides with the 0^{++} glueball mass in the strong coupling expansion [8]. If this result is valid also in the weak coupling limit, then the Abelian Dominance is reduced to the prediction that the dual photon mass m_V coincides with the 0^{++} glueball mass. Comparison of numerical results for the Yukawa mass μ with glueball masses can be found in [15].

Note that the prediction (67) is highly non-trivial in fact. Indeed the $|Q_M| = 1$ monopoles are so to say fundamental monopoles which look as Abelian monopoles at short distances and are associated for this reason with an infinite action. They are introduced therefore as external objects via the 't Hooft loop, similar to introduction of infinitely heavy quarks via the Wilson loop. The $|Q_M| = 2$ monopoles, on the other hand, have a finite action and their description as a fundamental objects seems to be granted only at large distances. This could be manifested, in particular, through existence of an intermediate region between the distances where the Coulombic and Yukawa pictures apply. In other words, the coefficient in front of the Coulombic term could have not matched the coefficient in front of the Yukawa-like

potential. However, existing data about the 't Hooft loop [15] indicate that the matching is exact, within the error bars. In other words, the dual Abelian Higgs model of QCD vacuum works already at smallest distances available on the lattice. Similar conclusions can be drawn in fact from the studies of the heavy quark potential induced by monopoles [21] and from description of the structure of the flux string [20, 22], for a review see [23].

5.3 Finite Temperatures.

The authors of Ref. [15] have also performed numerical calculations of the 't Hooft loop at finite temperatures, and determined the dependence of the Yukawa mass μ on the temperature. To provide a theoretical framework for the behavior of the 't Hooft loop at high temperatures we can use again the idea of the Abelian Dominance.

In more detail, we estimate the screening mass μ using the fact that the Abelian model which corresponds to the high temperature SU(2) gluodynamics is the 3D compact U(1) theory. Therefore the intermonopole potential at high temperatures is essentially given by (67,68), with μ now being the Debye mass [24]:

$$m_D^2 \;=\; 16\pi\,\frac{\rho}{e_3^2}\,, \tag{69}$$

where ρ is the density of Abelian monopoles and e_3 is the corresponding three-dimensional coupling constant. To estimate the temperature dependence of m_D we use the numerical results of Ref. [25], where the density of Abelian monopoles was obtained[3]:

$$\rho = 2^{-7}(1 \pm 0.02)\, e_3^6\,, \tag{70}$$

Therefore

$$m_D = 1.11(2)\, e_3^2\,. \tag{71}$$

Moreover, at high temperatures we can use the dimensional reduction formalism and express the 3D coupling constant e_3 in terms of the 4D Yang–Mills coupling g. At the tree level one has

$$e_3^2(T) = g^2(\Lambda, T)\, T\,, \tag{72}$$

where $g(\Lambda, T)$ is the running coupling calculated at the scale T,

$$g^{-2}(\Lambda, T) = \frac{11}{12\pi^2}\log\Bigl(\frac{T}{\Lambda}\Bigr) + \frac{17}{44\pi^2}\log\Bigl[2\log\Bigl(\frac{T}{\Lambda}\Bigr)\Bigr]\,, \tag{73}$$

and Λ is a dimensional constant which can be determined from lattice simulations.

At present the lattice measurements of the Λ parameter are not very precise. We use the results of two particular calculations. Namely, in Ref. [26] the lattice

[3] Note that the original result of Ref. [25] for the lattice monopole density is: $\rho_{lat.} = 0.50(1)\,\beta_G^3$, where β_G^3 is a three dimensional coupling constant which is expressed in terms of the 3D electric charge e_3 and lattice spacing a as $\beta_G^3 = 4/(a\, e_3^2)$. The physical density ρ of monopoles is given by $\rho = \rho_{lat.}\, a^{-3}$ which can easily be transformed into Eq. (70).

$T/\sqrt{\sigma}$	$\mu/\sqrt{\sigma}$	$m_D/\sqrt{\sigma}$	
		$\Lambda = 0.197\sqrt{\sigma}$	$\Lambda = 0.057\sqrt{\sigma}$
0.460	3.24(42)	5.12	2.04
1.225	4.13(41)	6.16	3.82
1.838	5.43(59)	7.67	5.12
3.676	17.3(4.1)	11.97	8.69

Table 1: The screening mass μ (see (67,68)) at different temperatures, Ref. [15], and our predictions for m_D obtained with different Λ, Eqs. (74,75).

data for the gluon propagator have been used to determine the so–called "magnetic mass" in high temperature SU(2) gluodynamics. These measurements imply the following value of Λ:

$$\Lambda = 0.262(18)\, T_c = 0.197(14)\, \sqrt{\sigma}\,, \tag{74}$$

where T_c is the temperature of the deconfinement phase transition, $T_c \approx 0.75\sqrt{\sigma}$. In Ref. [27], on the other hand, the spatial string tension has been calculated and the corresponding value of Λ turned to be three times smaller:

$$\Lambda = 0.076(13)\, T_c = 0.057(10)\, \sqrt{\sigma}\,. \tag{75}$$

Collecting Eqs. (71)-(75) we get predictions for the Debye mass which are shown in Table 1 along with the values of mass μ obtained numerically in Ref. [15]. One can clearly see that the predictions and the numerical results are in agreement within the theoretical uncertainties. There are at least three sources of these uncertainties. First, the value of Λ is not determined precisely as we already noted. Second, we have used the dimensional reduction which is supposed to work well only at asymptotically high temperatures, while only one value $T = 3.676\,\sqrt{\sigma} \approx 5\,T_c$ in the Table 1 may be considered as high enough. Third, as we already noted the lattice and continuum Yukawa interactions are substantially different. For example, we may treat the 't Hooft loop quantum average studied in Ref. [15] as a two–point correlator in three spatial dimensions. Then we may use the results of Ref. [28] and relate the value of μ obtained with the use of the continuum propagator to the correct value, $\mu^{\mathrm{correct}} \approx \frac{2}{a}\mathrm{ArcSinh}\left(\frac{\mu a}{2}\right)$. If we apply this correction to the values of μ in Table 1 then for $a\mu = 2.29(55)$ and $T = 3.676\,\sqrt{\sigma}$ the correction is essential. Indeed, we obtain: $\mu^{\mathrm{correct}} \approx 14.8(3.5)$, which is quite close to our prediction with $\Lambda = 0.197\sqrt{\sigma}$.

Conclusions

We have tried to formulate a theoretical framework which would allow for monopoles in the continuum version of non-Abelian gauge theories. Indeed, monopoles nowadays are very common field configuration on the lattice. In the continuum, on the other hand, monopoles appear to be associated with singular fields and divergent action.

The key element to introduce monopoles in the continuum is to allow for Dirac strings. While naively the action associated with the Dirac strings is infinite, they cost no action at all in the compact U(1) gauge model [4]. Thus, within the continuum formulation, one has to postulate that there are certain singular fields which cost no action as well. An alternative representation for the singular fields are Dirac sheets (see, e.g., Eq. (8) above). In the non-Abelian case, we argued that the continuum version should admit certain singular or stringy fields without any change in the action. One can say that the Dirac strings which cost no action are aligned in the color space with the background, or regular fields.

Once the Dirac strings are admitted into the continuum version of gluodynamics, the end points of the strings, or monopoles, cost in the perturbative vacuum no action either. This is true both classically and with account of quantum corrections. And this is in distinction from the U(1) case where the end points are monopoles with an ultraviolet divergent action. As a result, although the modified continuum version appears very different from the standard one since it allows for singular potentials inversely proportional to the coupling, perturbatively the two theories are in fact equivalent. Thus, at this point the problem seems to be the other way around. Namely, there is no difficulty any longer to introduce fields which look as monopoles in terms of Abelian fields but cost no action and appear as gauge artifacts once the full spectrum of the non-Abelian degrees of freedom is taken into account.

The difference between the two formulations becomes manifest once the gauge is fixed a la 't Hooft [5] and background non-perturbative fields are introduced. The point is that in presence of the background field only those Dirac strings which are parallel to the background in the color space are non observable. On the other hand, the definition of the monopoles in terms of the topology of the gauge fixing introduces Dirac strings which do not satisfy this condition. As a result, the action associated with the monopoles is not vanishing any longer. And the monopoles do emerge as possible fluctuations with finite action which are present in the continuum theory modified to incorporate Dirac strings. It is worth emphasizing that upon integration over the background fields the monopole action does not depend on the position of the Dirac string but only on the monopole trajectory.

The machinery to prove the independence on the position of the Dirac string is also all what is needed to introduce a continuum analog of the 't Hooft loop operator [6]. The continuum formulation of the 't Hooft loop is one of the central points of this paper. Furthermore, we were able to derive both rigorous and model-dependent results for the behavior of the 't Hooft loop at zero and high temperature SU(2)

gluodynamics.

In terms of physical applications, the picture developed explains in generic terms correlation between instantons and monopoles (for discussion see, e.g., [11]). Also, it was demonstrated that while perturbatively the modified theory allowing for the Dirac strings is equivalent to the standard one, non-perturbatively they are different. This might explain a kind of mystery with the non-perturbative $1/Q^2$ corrections from short distances which seem to exist phenomenologically but evade, so far, theoretical understanding within the standard framework (for reviews and further references see [29]).

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